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MDE/M-23 **4077**

ADVANCED ABSTRACT ALGEBRA

Paper–I: MM–401

Time Allowed : 3 Hours] [Maximum Marks : 80

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. All questions carry equal marks.

UNIT–I

1. State and prove Zassenhaus's Lemma.

2. (a) Prove that S_3 is Solvable group.

   (b) Prove that a group of order 7^3 is Nilpotent.

UNIT–II

3. If E is finite extension of F, prove that E is an algebraic extension of F. If converse true ? Justify your claim.

4. (a) Prove that the multiplicative group of non zero elements of a finite field is cyclic.

   (b) Prove that $\mathbb{Q}\left(\sqrt{2}, \sqrt{3}\right) = \mathbb{Q}\left(\sqrt{2} + \sqrt{3}\right)$.

5. (a) Prove that the Galois group of $x^4 - 2 \in \mathbb{Q}[x]$ is the group of symmetries of a square.

   (b) Prove that the regular 17-gon is constructible with ruler and Compass.

UNIT–III

6. If $T \in A(V)$ has all its characteristic roots in F, prove that there is a basis of V in which the matrix of T is triangular. Also, prove that T satisfies a polynomial of degree n over F if V is n-dimensional over F.

7. Prove that two Nilpotent linear transformations are similar if and only if they have the same invariants.

UNIT–IV

8. (a) Let M be a free R-module with a basis $\{e_1, e_2, ..., e_n\}$. Prove that $M \simeq R^n$.

   (b) State and prove Schur's Lemma.

9. State and prove Noether-Lasker theorem.

10. (a) Obtain the Smith normal form and rank for the matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \end{bmatrix}.$$

   (b) Find the abelian group generated by $\{x_1, x_2, x_3\}$ subject to:

$$5x_1 + 9x_2 + 5x_3 = 0$$

$$2x_1 + 4x_2 + 2x_3 = 0$$

$$x_1 + x_2 - 3x_3 = 0.$$